Macroscopic coherent dressing of Bose polarons

Pietro Massignan

UPC
UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

ICFO
Institut de Ciències Fotòniques

Collaborators

Nils Guenther
(ICFO)

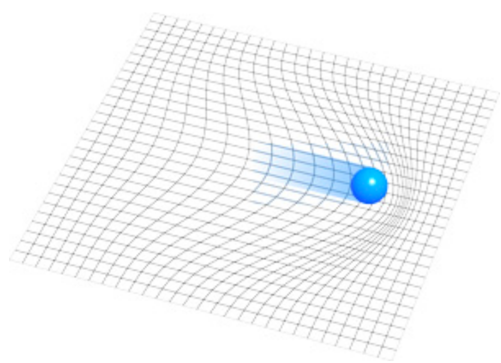
Maciej Lewenstein
(ICFO)

Richard Schmidt
(MPQ - Munich)

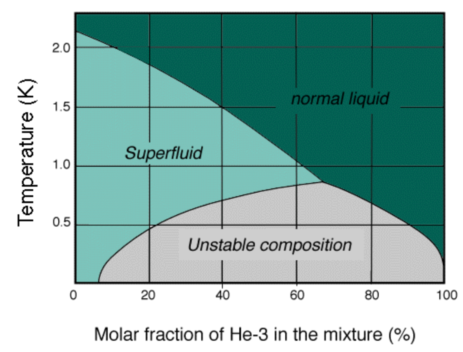
Georg Bruun
(Aarhus)

Victor Gurarie
(Boulder)

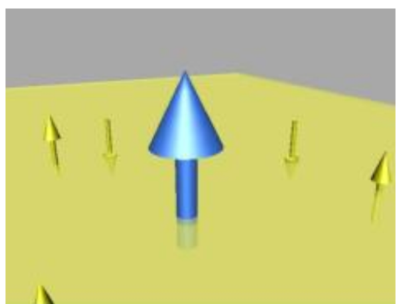
Dilute impurities in quantum many-body systems



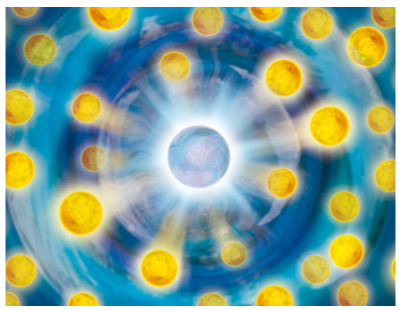
electrons
in a phonon bath



^{3}He in ^{4}He



magnetic impurities
(Kondo physics)



ultra-dilute
quantum mixtures

MIT [PRL ’09, PRL’19, arXiv 2019]
ENS [PRL ’09]
Innsbruck [Nature 2012, Science 2017]
Cambridge [Nature 2012]
LENS [PRL 2018],
Aarhus [PRL 2016]
JILA [PRL 2016]

✦ dressed impurities: simplest building block in a bottom-up approach to many-bodies

✦ ideal probes of equilibrium and non-equilibrium properties (transport, topology, …)

✦ well-posed problem (QMC, variational, diagrams, RG, functional determinants, …)

Recent reviews: PM, Zaccanti and Bruun [Rep. Prog. Phys 2014]
Schmidt, Knap *et al.* [Rep. Prog. Phys. 2018]

Outline

✦ Impurities in a weakly-interacting Bose gas ("Bose polarons")

1. Introduction

2. A *bosonic orthogonality catastrophe* arises in ideal BECs

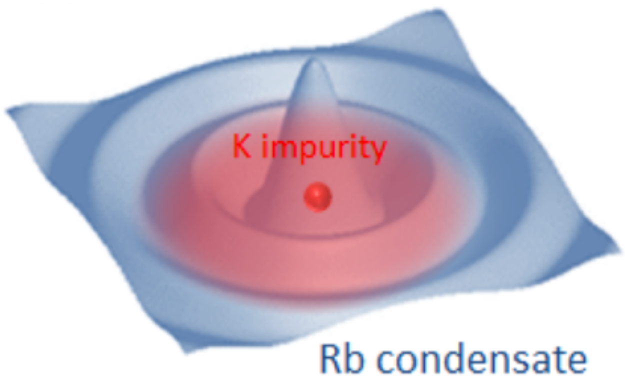
3. Mobile impurities in a weakly-interacting BEC: a simple Ansatz does marvels

Part I

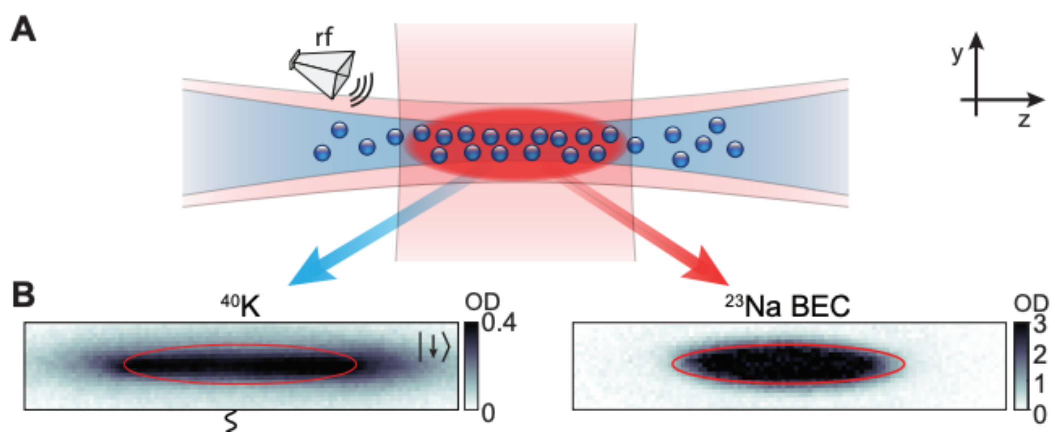
Introduction

Impurities in a Bose gas

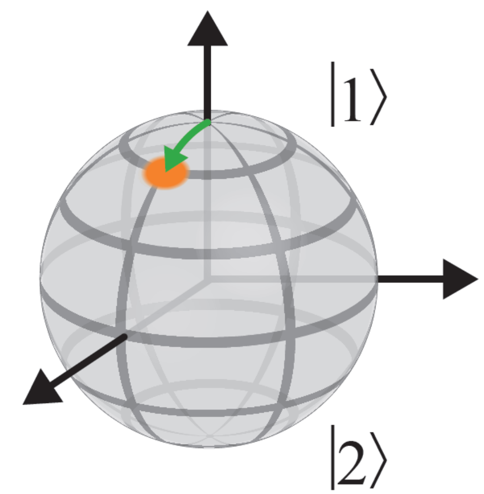
JILA: Hu, …, Cornelll and Jin [PRL 2016]



MIT: Yan, …, Zwierlein [arXiv 2019]



Aarhus: Jørgensen, …, Bruun and Arlt [PRL 2016]



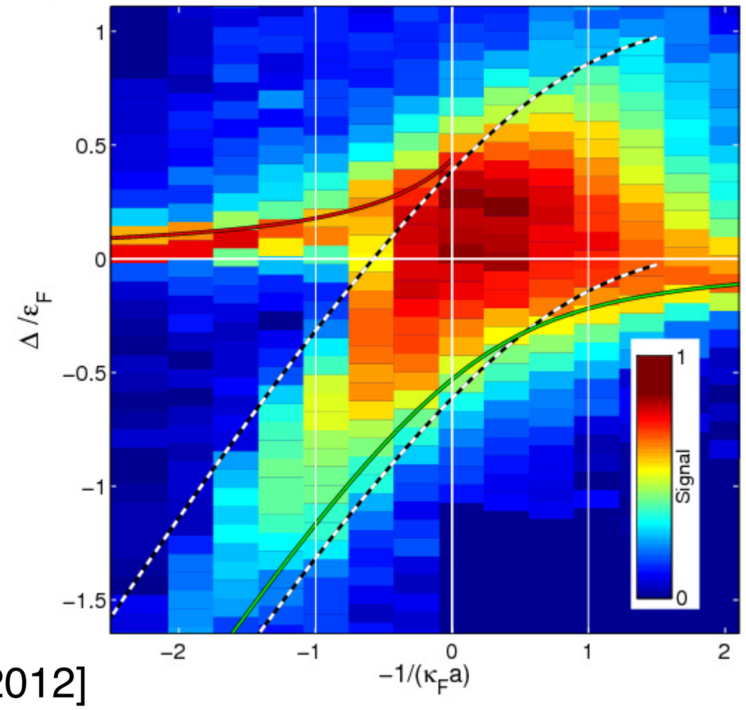
K gas spin-polarized in state $|1\rangle$
+ weak RF pulse
+ quick decoherence

= a few $|2\rangle$ impurities in a bath of $|1\rangle$ atoms

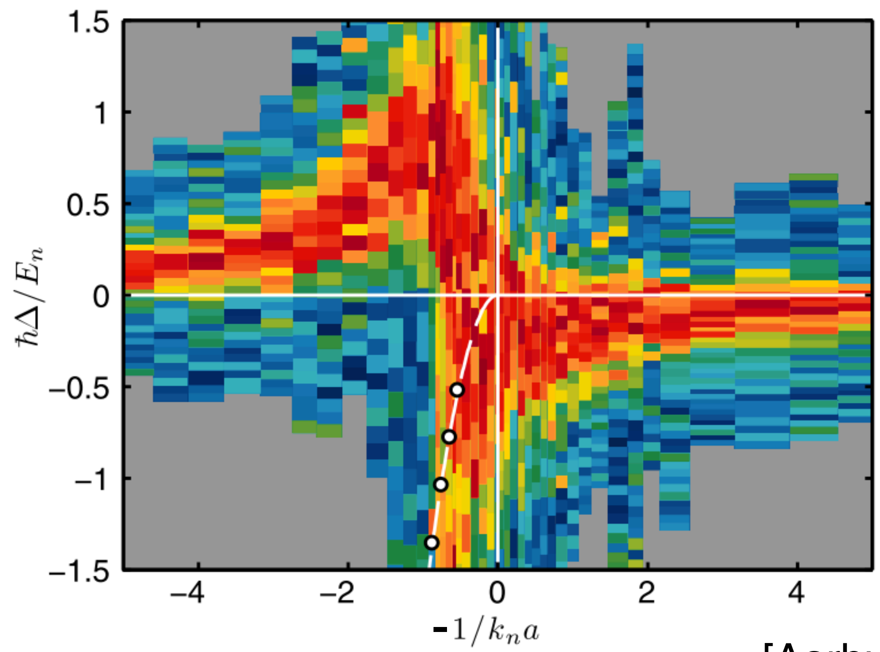
Theory: Schmidt, Das Sarma, Bruun, Levinsen, Parish, Demler, Camacho-Guardian, Pohl, Zhai, Cui, …

QMC: Peña-Ardila, Giorgini, Astrakharchik, …

Polarons:
Fermi vs. Bose



[Innsbruck 2012]

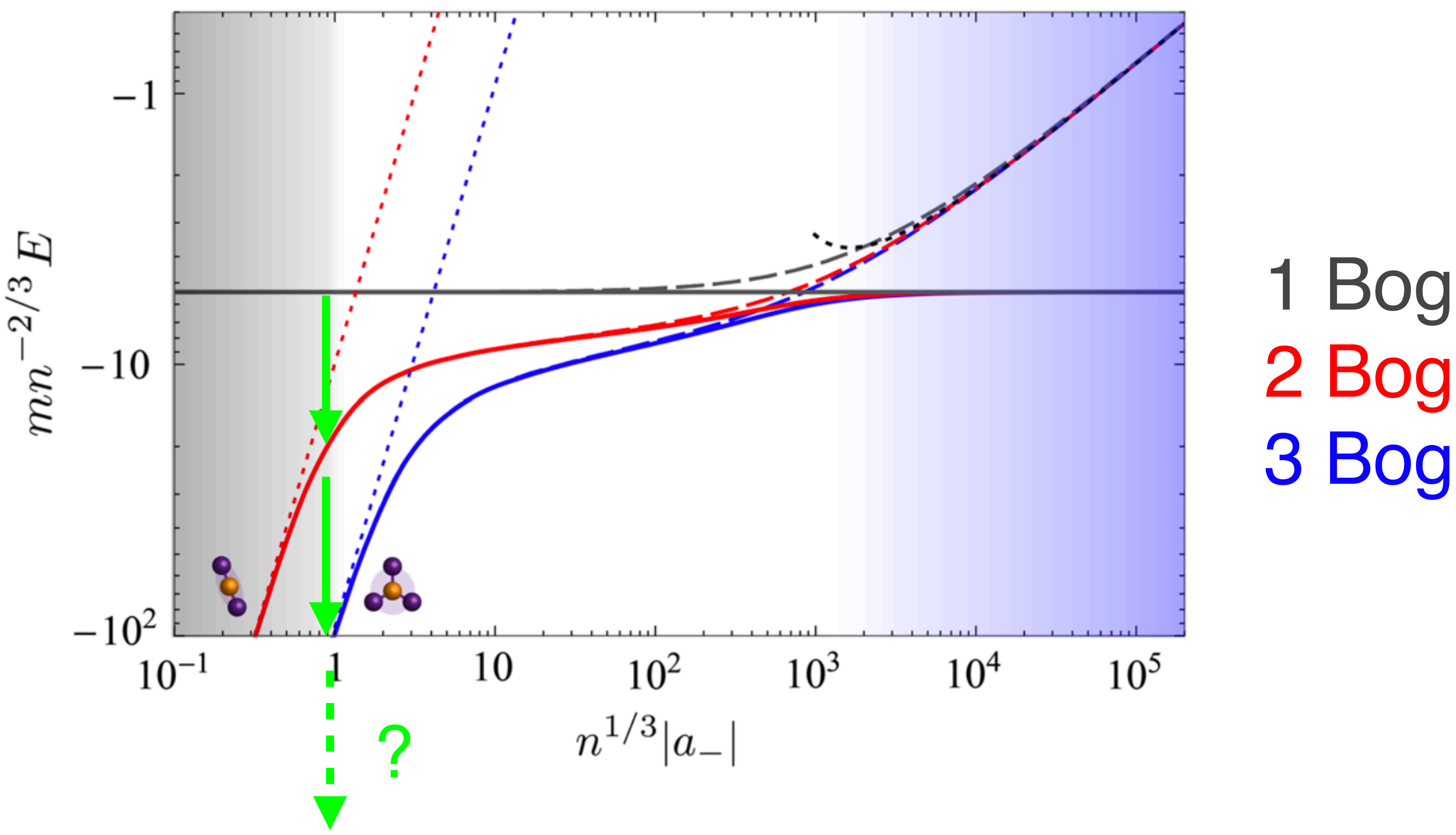


[Aarhus 2016]

Differences	non-interacting Fermi sea	weakly-interacting Bose gas
Bath excitations	particle/hole	Bogolubov modes
Ground state (polaron/molecule)	sharp transition	smooth crossover
Stability	(meta-)stable mixture	rapid three-body losses
Few - body physics	negligible	important
Temperature	smooth crossover from degenerate to classical	BEC phase transition

Accuracy of the "Chevy Ansatz" for Bose polarons

Polaron energy at unitarity



Yoshida, Endo, Levinsen and Parish, PRX 2018

Part II

Static impurity inside an ideal BEC

$$m_I = \infty \qquad \& \qquad a_B = 0$$

A static impurity in an ideal BEC

- 1 boson inside a sphere of radius R : $\psi_0(r) \propto \sin(k_0 r)/r$
- Add a short-ranged interaction with an impurity at $r = 0$: $\psi_1(r) \propto \sin(k_1 r + \delta)/r$
- Phase shift: $k \cot \delta = -1/a + O(k^2)$
- Ideal BEC with N bosons: $|\Psi\rangle = \otimes^N |\psi\rangle$
- Energy shift: $\Delta E = N \dfrac{\hbar^2(k_1^2 - k_0^2)}{2m_B} = \dfrac{2\pi\hbar^2}{m_B}\left(-\dfrac{\delta}{k}\right) n_0$

m_B : boson mass

- Weak interaction: $\delta \approx -ka$ ⇨ usual mean-field shift $\Delta E \propto a$

PM, Pethick and Smith, Phys. Rev. A **71**, 023606 (2005)

A static impurity in an ideal BEC

◉ How close are the final and initial states is quantified by the residue:

$$Z = |\langle \Psi_0 | \Psi_1 \rangle|^2 = |\langle \psi_0 | \psi_1 \rangle|^{2N} \approx e^{-\alpha N^{1/3}(k_n a)^2}$$

$\alpha = 0.17$

$k_n = (6\pi n_0)^{1/3}$

◉ For large N, the residue vanishes for every impurity-bath interaction $a \neq 0$

◉ A static impurity + ideal BEC ➜ bosonic orthogonality catastrophe

Part III

Mobile impurity inside a weakly-interacting BEC

$m_I < \infty \qquad \& \qquad a_B > 0$

The model

- Grand potential:

bath BEC, impurity, impurity-bath interaction

$$\hat{\Omega} = \int d\mathbf{r} \left[\hat{b}^\dagger_\mathbf{r} \left(-\frac{\hbar^2 \nabla^2_\mathbf{r}}{2m_B} + \frac{2\pi\hbar^2 a_B}{m_B} \hat{b}^\dagger_\mathbf{r} \hat{b}_\mathbf{r} - \mu \right) \hat{b}_\mathbf{r} + \hat{c}^\dagger_\mathbf{r} \left(-\frac{\hbar^2 \nabla^2_\mathbf{r}}{2m_I} \right) \hat{c}_\mathbf{r} + \int d\mathbf{s}\ \hat{b}^\dagger_\mathbf{r} \hat{b}_\mathbf{r} U(\mathbf{r}-\mathbf{s}) \hat{c}^\dagger_\mathbf{s} \hat{c}_\mathbf{s} \right]$$

at fixed asymptotic density n_0 —> fixed chemical potential $\mu = \frac{4\pi\hbar^2 a_B}{m_B} n_0$

- Coherent states for the bath: $|\phi(\mathbf{r})\rangle \equiv e^{\int d\mathbf{s}[\phi(\mathbf{r}-\mathbf{s})\hat{b}^\dagger_\mathbf{s} - \mathrm{c.c.}]} |0\rangle$

- These satisfy $\hat{b}_\mathbf{s} |\phi(\mathbf{r})\rangle = \phi(\mathbf{r}-\mathbf{s}) |\phi(\mathbf{r})\rangle$

- Directly related to the impurity-bath density-density correlation function:

$$\langle \hat{n}_I(\mathbf{r}) \hat{n}_B(\mathbf{r}+\mathbf{s}) \rangle_\mathbf{r} \sim |\phi(\mathbf{s})|^2$$

Modified GPE

- Variational coherent Ansatz: $|\Psi\rangle \equiv \int \mathrm{d}\mathbf{r}\, \hat{c}^\dagger_{\mathbf{r}}\, |\phi(\mathbf{r})\rangle\, |0\rangle$

- Minimizing $\hat{\Omega}$ w.r.t. $|\Psi\rangle$ gives a *modified Gross-Pitaevskii equation*:

$$\left[-\frac{\hbar^2 \nabla^2_{\mathbf{r}}}{2m_r} + U(\mathbf{r}) + \frac{4\pi\hbar^2 a_B}{m_B}\,|\phi(\mathbf{r})|^2\right] \phi(\mathbf{r}) = \mu\phi(\mathbf{r})$$

reduced mass: $m_r^{-1} = m_B^{-1} + m_I^{-1}$

- Far away from the impurity $U(\mathbf{r}) \to 0$ and $|\phi(\mathbf{r})|^2 \to n_0$

- Natural units:

coherence length $\xi = \dfrac{1}{\sqrt{8\pi n_0 a_B m_r / m_B}}$ and energy $E_\xi = \dfrac{\hbar^2 n_0 \xi}{2m_r}$

Modelling the impurity-bath interaction

- Choose an explicit model potential for $U(\mathbf{r})$:

  - short-ranged

  - radially-symmetric

  - square-well, exponential, gaussian, …

- For $|a| \ll \xi$, linearize the mod-GPE to find the universal Yukawa behavior:

$$\phi(r) = \sqrt{n_0}\left(1 - \frac{a}{r}e^{-\sqrt{2}r/\xi}\right)$$

- For stronger interactions, challenging numerics because $r_e \sim 0.001\xi$

Universality?

- Bad surprise:

  - for stronger interactions, results strongly depend on the choice of $U(r)$

  - the mod-GPE does not admit a well-defined zero-range limit…

- What to do?

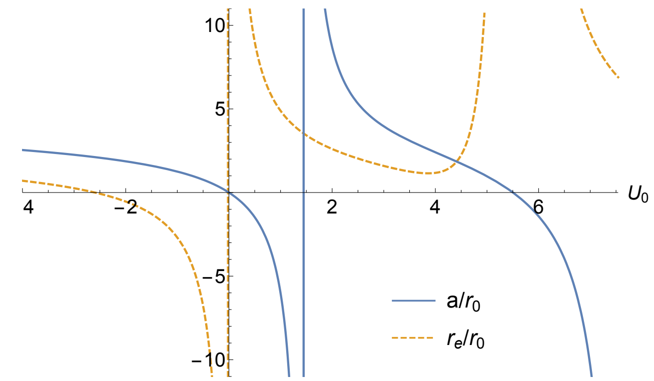
- Low-energy scattering: $k \cot \delta = -1/a + r_e k^2/2 + o(k^2)$

- Results become model-independent if one fixes both a and r_e

- Two-parameter universality

Effective range??

- May be computed numerically, from the solution of the two-body problem



- But in experiments?

- At resonance: $r_e = 2.79R_{\mathrm{vdW}} - 2R^*$

  - open-channel dominated (broad): $r_e > 0$

  - closed-channel dominated (narrow): $r_e < 0$

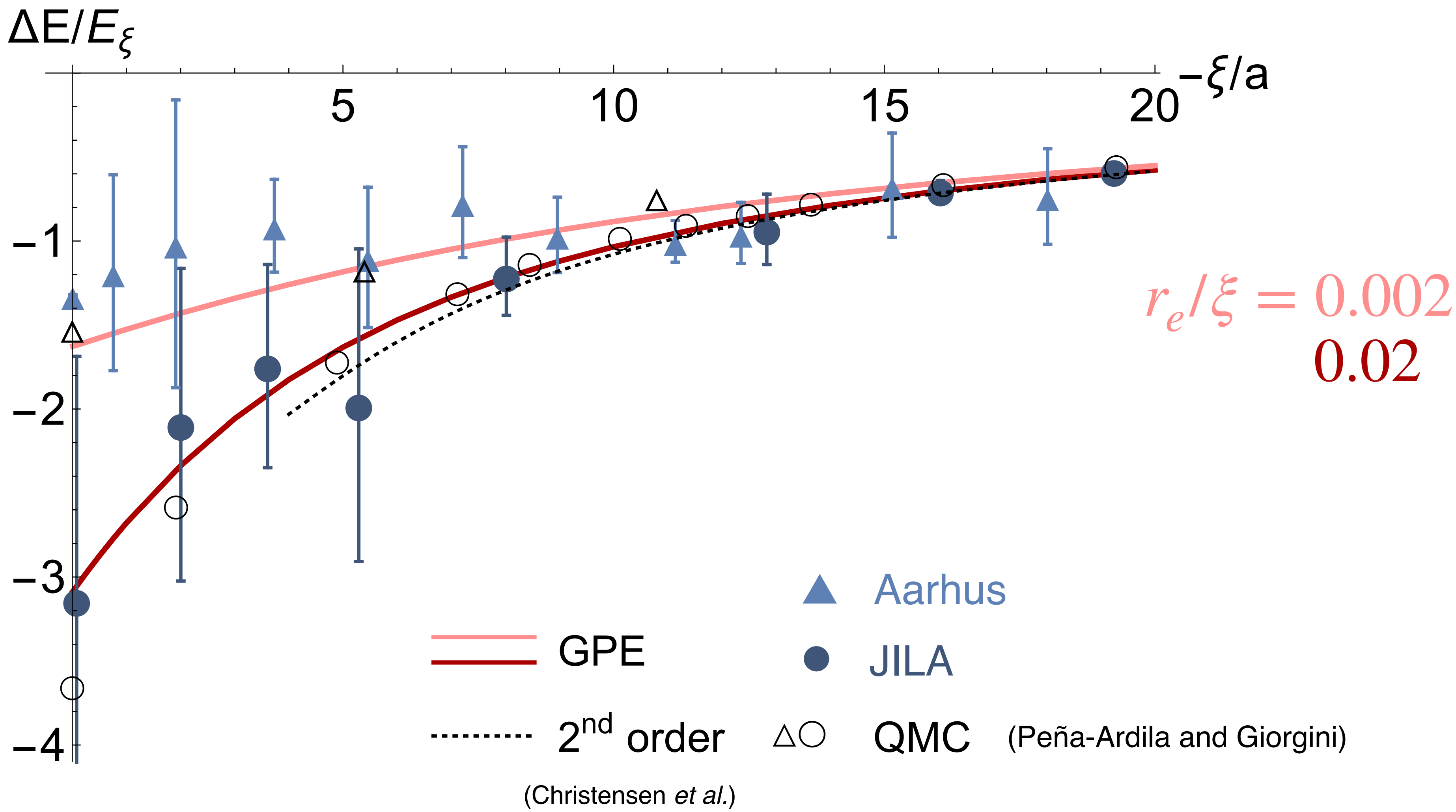
$$R_{\mathrm{vdW}} \sim 100a_0$$

$$R^* = \frac{\hbar^2}{2m_r a_{\mathrm{bg}}\,\delta\mu\,\Delta B} > 0$$

	$r_e\ (a_0)$	r_e/ξ	m_I/m_B
Aarhus	36	0.002	1
MIT	137	0.01	40/23
JILA	131	0.02	40/87

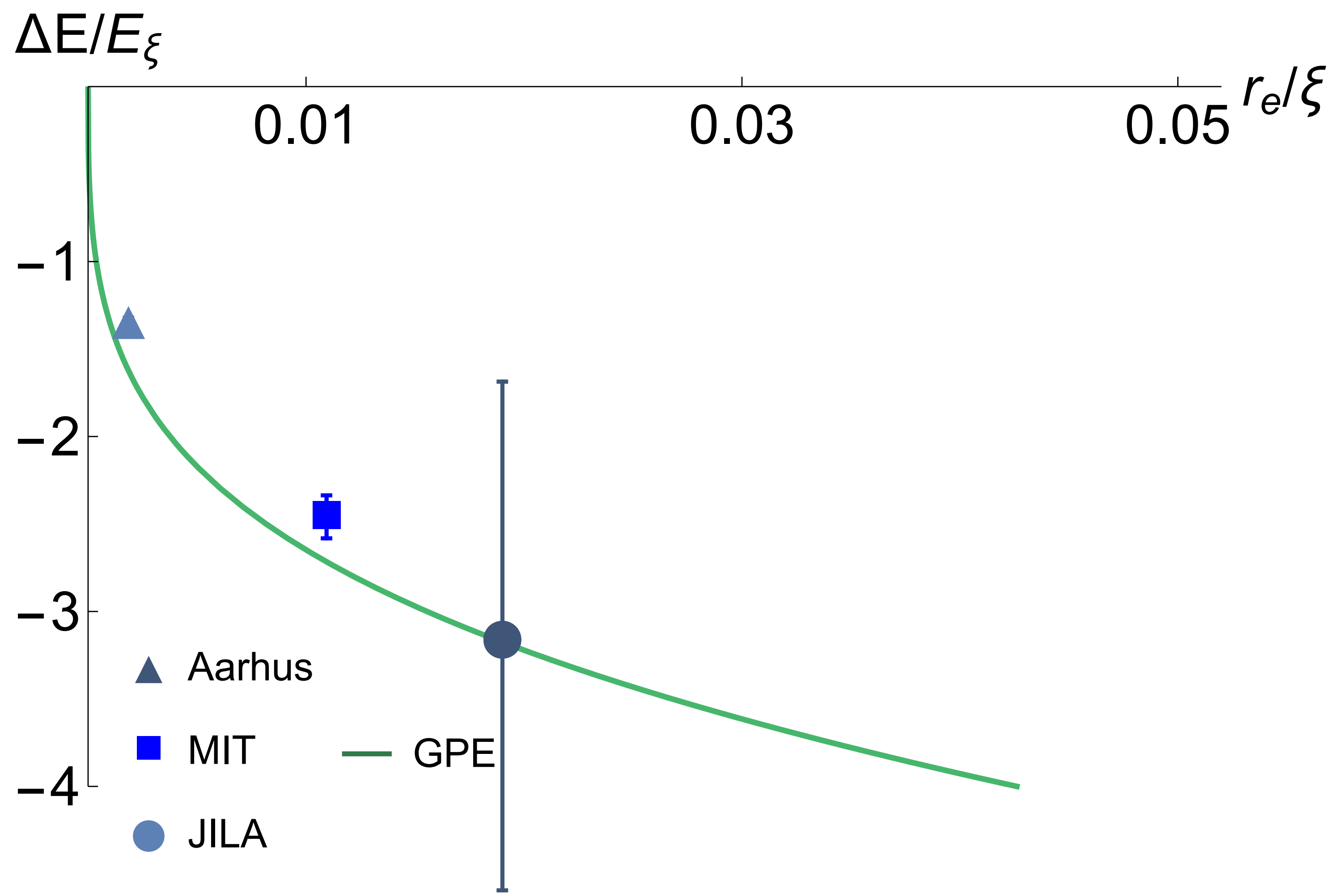
Viel and Simoni, Phys. Rev. A **93**, 042701 (2016)
Tanzi, Cabrera, Sanz, Cheiney, Tomza and Tarruell, Phys. Rev. A **98**, 062712 (2018).

Polaron energy



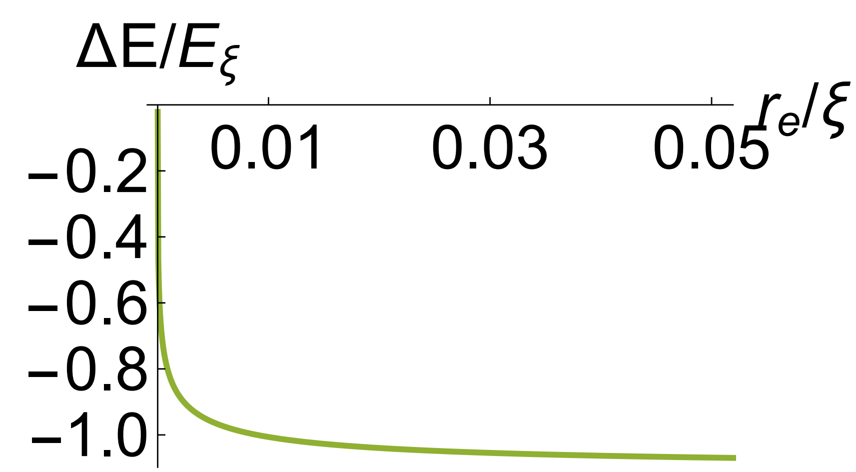
Polaron energy vs. range

- Strong range-dependence at unitarity:

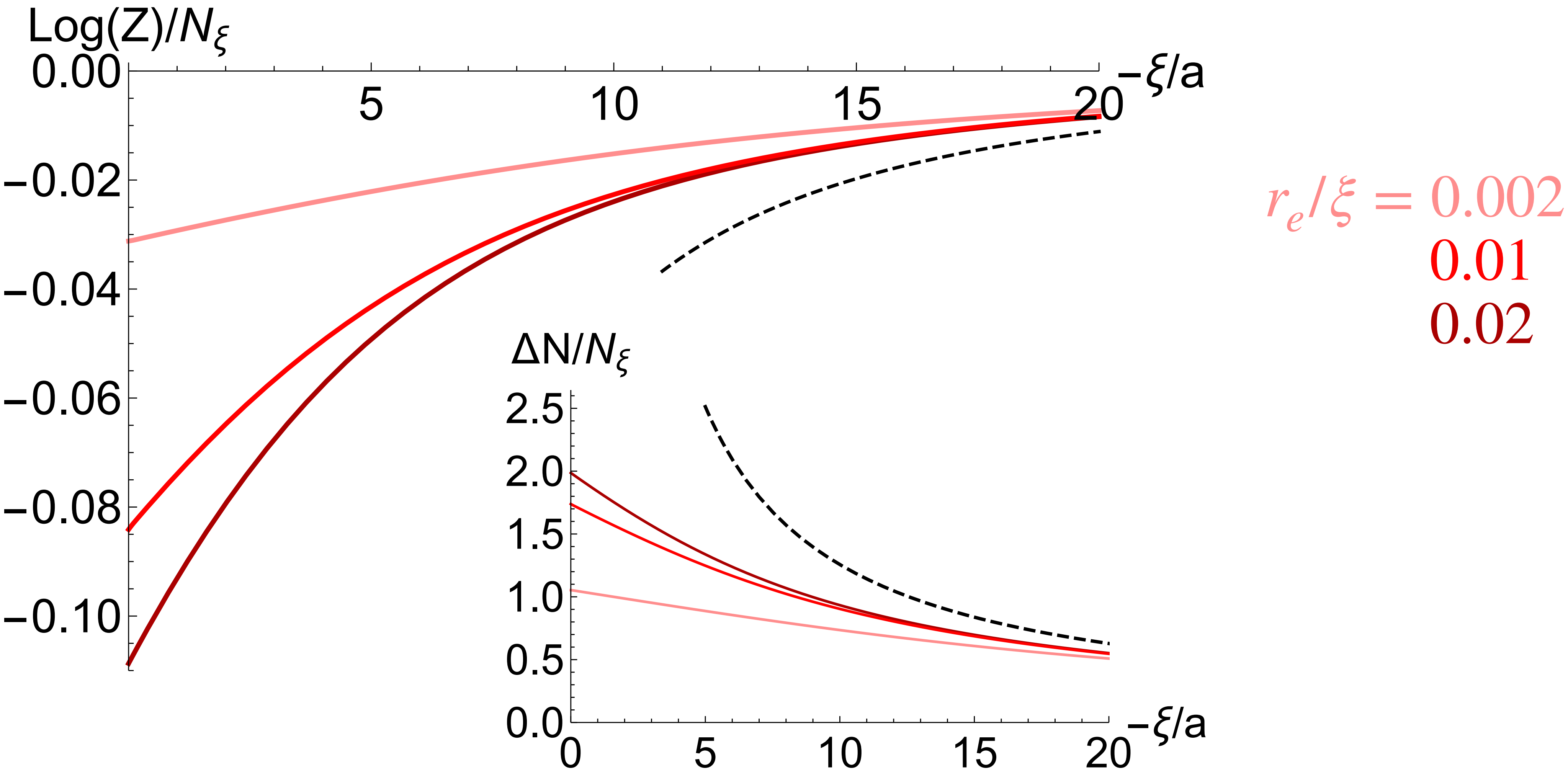


- Much weaker dependence at weaker attraction:

$$(-\xi/a = 10)$$



Residue and ΔN

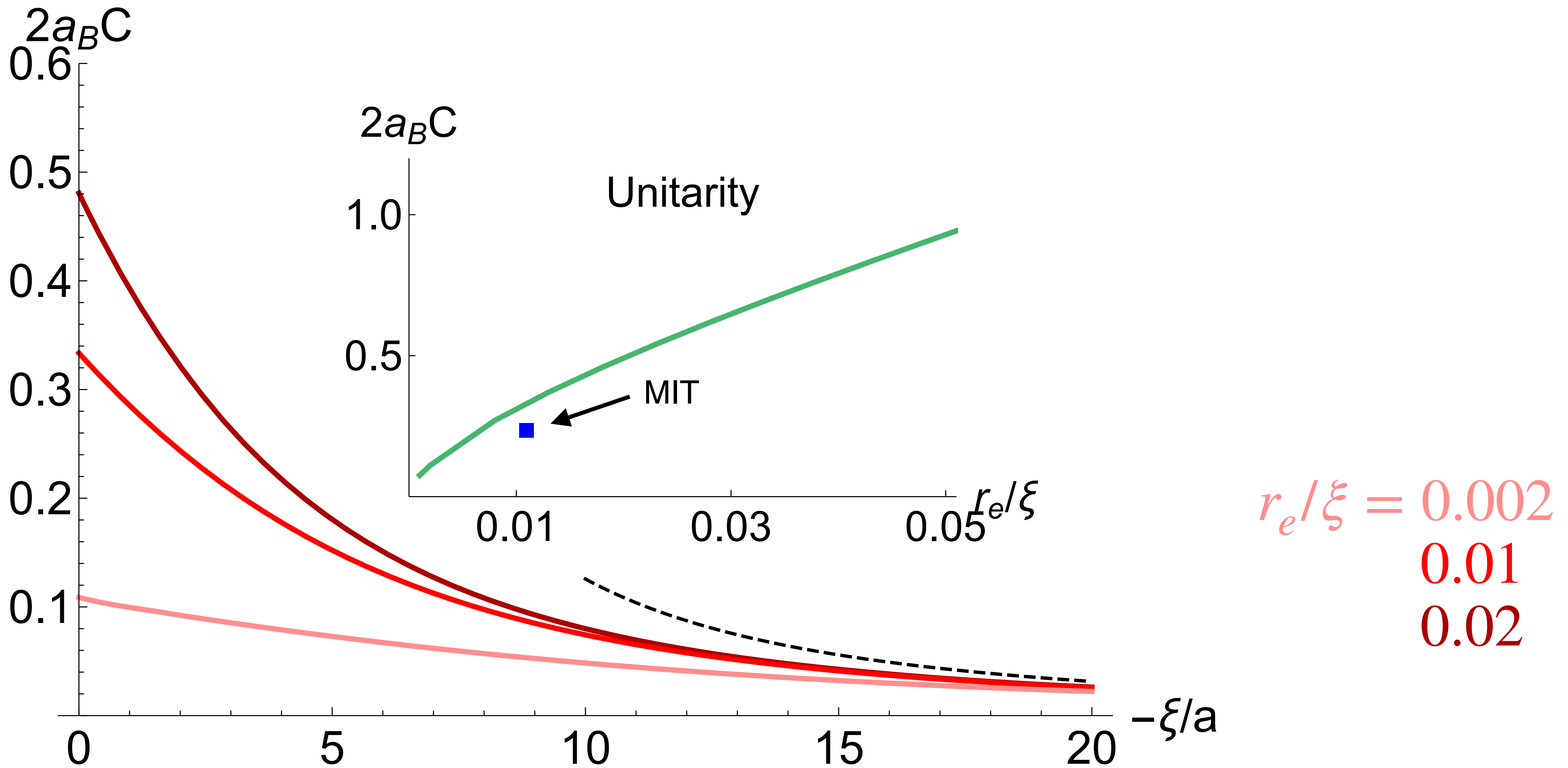


Number of BEC atoms in the dressing cloud: $\Delta N = \int \mathrm{d}\mathbf{r}\, [n(\mathbf{r}) - n_0]$

Perturbative results: $\begin{cases} \log(Z) = -\sqrt{2}\pi n_0 \xi a^2 \\ \Delta N = -a m_r/(2 a_B m_B) \end{cases}$

Tan's contact

$$C = -\frac{8\pi m_r}{\hbar^2}\frac{\partial(\Delta E)}{\partial(a^{-1})}$$



Perturbative result: $C = 16\pi^2 n_0 a^2$

How reliable is all this?

- The (modified) GPE is accurate provided the gas parameter na_B^3 is small

- Close to the impurity the density rises rapidly.

  Breakdown?

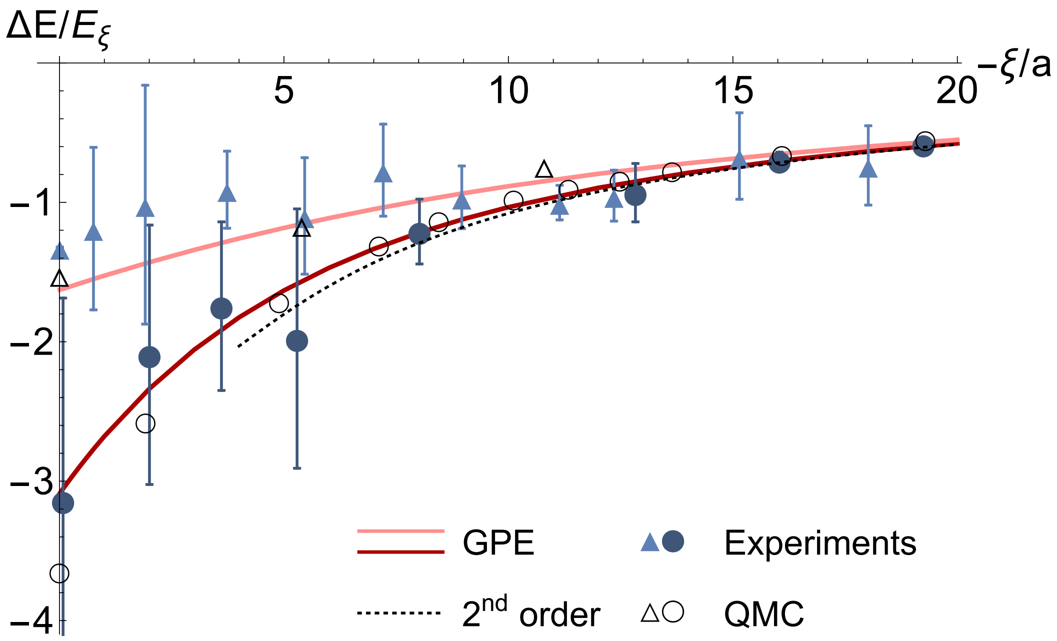
- Numerics: the gas parameter remains **small everywhere** $\left[n(\mathbf{r})a_B^3 < 7 \cdot 10^{-3}\right]$

- Ideal gas limit: $\begin{pmatrix} a_B \to 0 \\ \text{fixed } r_e \end{pmatrix}$

$$\begin{cases} n(0)a_B^3 \to 0 & \text{dilute gas} \\ \Delta E \to -\infty & \\ \Delta N \to +\infty & \text{macroscopic dressing} \\ Z \to 0 & \text{recovers AOC} \end{cases}$$

Conclusions

- Bose polarons greatly differ from Fermi ones
- Crucial role played by large compressibility of the weakly-interacting BEC
- Non-perturbative treatment of the BEC deformation is crucial
- A *bosonic orthogonality catastrophe* arises when $a_B \to 0$
- An accurate variational Ansatz predicts macroscopic coherent dressing cloud
- **A simple theory in remarkable agreement with all experimental and QMC data**



Thank you!